NVIDIA

East Building, Ballroom BC

nvidia.com/siggraph2018

Machine Learning and Rendering

Alex Keller, Director of Research

Machine Learning and Rendering

Course web page at `https://sites.google.com/site/mlandrendering/`

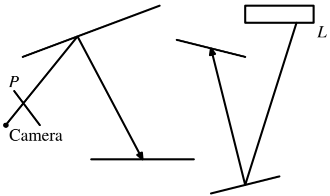
- 14:00 From Machine Learning to Graphics and back
  - Alexander Keller, NVIDIA
- 14:40 Robust & Efficient Light Transport by Machine Learning
  - Jaroslav Křivánek, Charles University, Prague
- 15:15 Deep Learning for Light Transport Simulation
  - Jan Novàk, Disney Research
- 16:05 Neural Realtime Rendering in Image Space
  - Anton Kaplanyan, Facebook Reality Labs
- 16:40 Deep Realtime Rendering
  - Marco Salvi, NVIDIA

Modern Path Tracing

Light transport simulation

- ways to formulate the radiance L_r reflected in a surface point x

$$L_r(x, \omega_r) = \int_{\mathscr{S}^2_-(x)} L_i(x, \omega) f_r(\omega_r, x, \omega) \cos\theta_x \, d\omega$$



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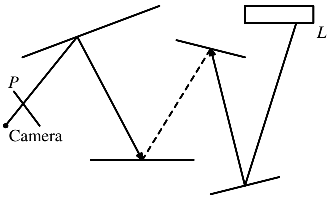

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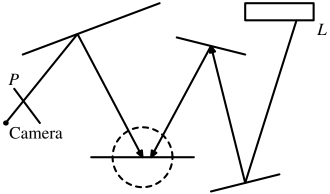

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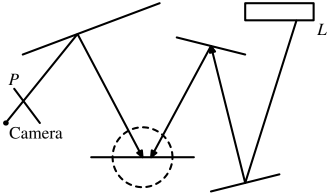

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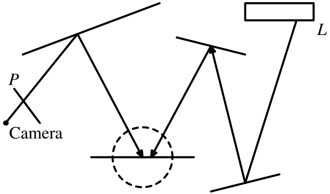

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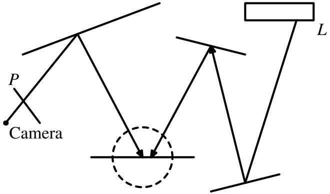

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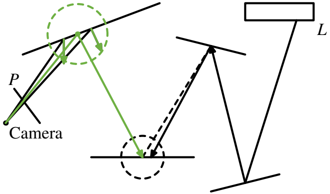

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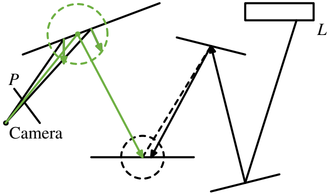

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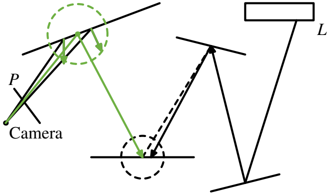

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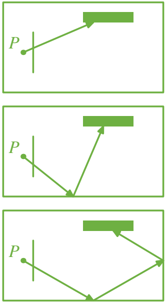
Modern Path Tracing

Light transport simulation

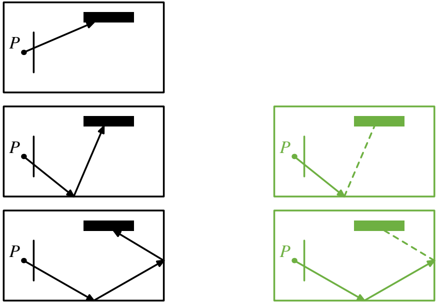
- path tracing: Starting paths from camera and iterating scattering and ray tracing
  - bad for small light sources, good for large light sources



Modern Path Tracing

Light transport simulation

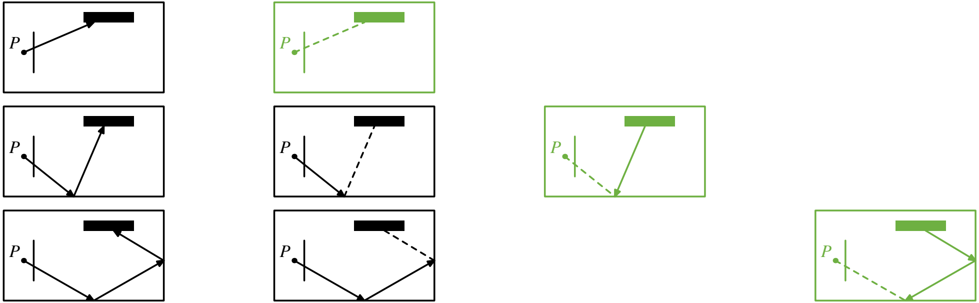
- path tracing with next event estimation by shadow rays (dashed lines)
  - good for small light sources, bad for close light sources



Modern Path Tracing

Light transport simulation

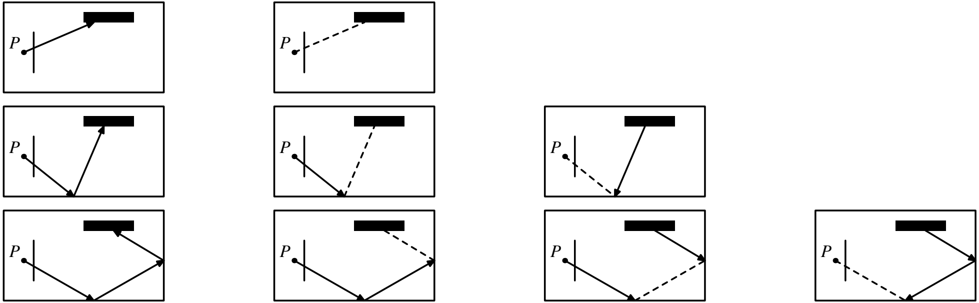
- light tracing, i.e. paths starting from the light source connected to the camera
  - can capture some caustics, where path tracing and next event estimation do not work



Modern Path Tracing

Light transport simulation

- all obvious ways to generate light transport paths
  - which ones are good ?



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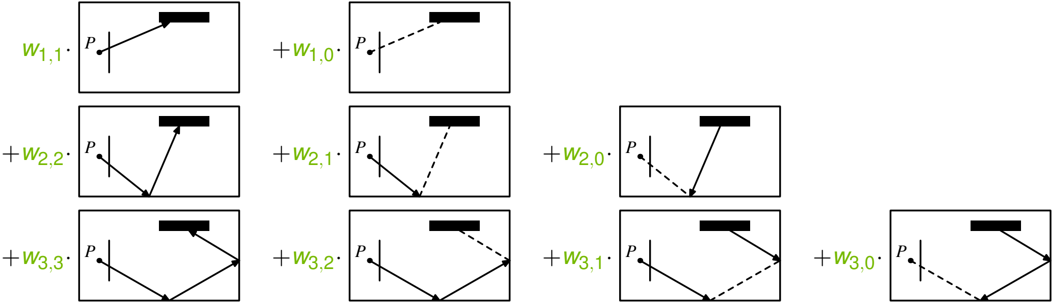
Modern Path Tracing

Light transport simulation

- bidirectional path tracing, optimally combining all techniques by weighting each contribution
  - $\sum_{i=0}^{l} w_{l,i} = 1$ for path length $l-1$, $l \in \mathbb{N}$



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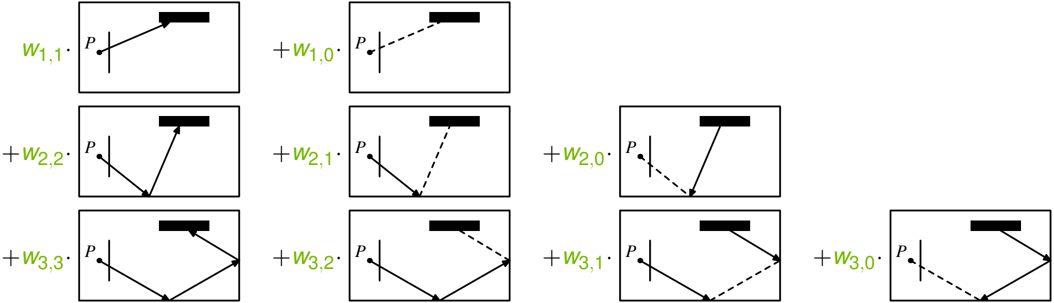

- problem of insufficient techniques, for example, if only one $w_{l,i} \neq 0$

Modern Path Tracing

Numerical integro-approximation

- Monte Carlo methods

$$g(y) = \int_{[0,1)^s} f(y,x)\,dx$$

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$$g(y) = \int_{[0,1)^s} f(y,x)dx \approx \frac{1}{n}\sum_{i=1}^{n} f(y,x_i)$$

  - uniform, independent, unpredictable random samples x_i
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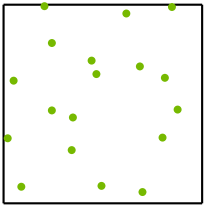

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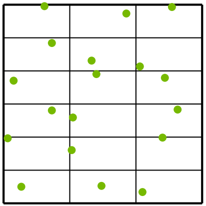

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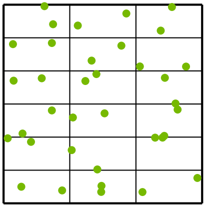

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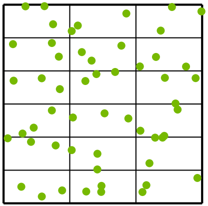

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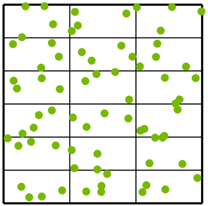

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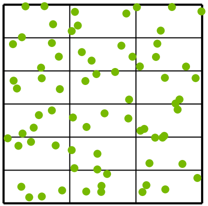

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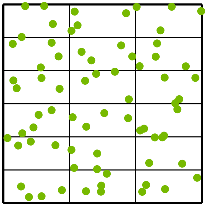

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  - much more uniform correlated samples x_i
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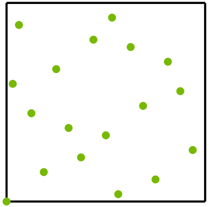

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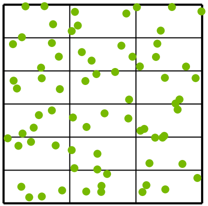

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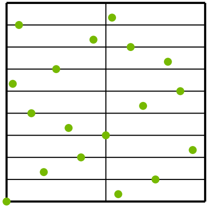

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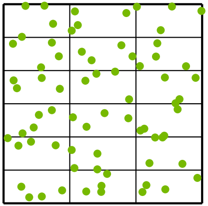

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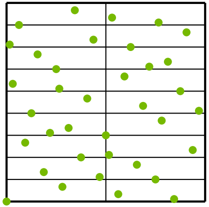

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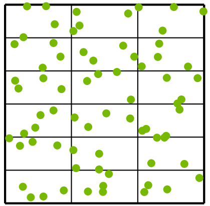

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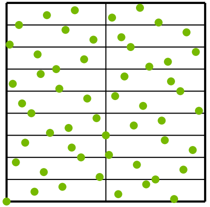

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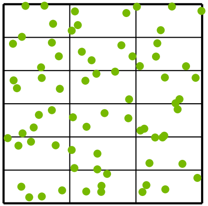

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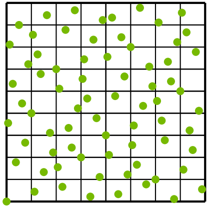

Modern Path Tracing

Pushbutton paradigm

- deterministic
  - may improve speed of convergence
  - reproducible and simple to parallelize

Modern Path Tracing

Pushbutton paradigm

- deterministic
  - may improve speed of convergence
  - reproducible and simple to parallelize
- unbiased
  - zero difference between expectation and mathematical object
  - not sufficient for convergence

Modern Path Tracing

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- deterministic
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  - reproducible and simple to parallelize
- biased
  - allows for ameliorating the problem of insufficient techniques
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► Quasi-Monte Carlo image synthesis in a nutshell
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▶ Quasi-Monte Carlo image synthesis in a nutshell
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Reconstruction from noisy input: Massively parallel path space filtering (link)

From Machine Learning to Graphics

Machine Learning

Taxonomy

- unsupervised learning from unlabeled data
  - examples: clustering, auto-encoder networks

Machine Learning

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- semi-supervised learning by rewards
  - example: reinforcement learning

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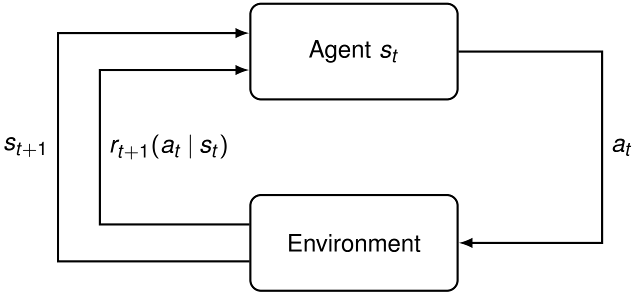
  - example: reinforcement learning
- supervised learning from labeled data
  - examples: support vector machines, decision trees, artificial neural networks

Reinforcement Learning

Goal: maximize reward

- state transition yields reward

$$r_{t+1}(a_t \mid s_t) \in \mathbb{R}$$



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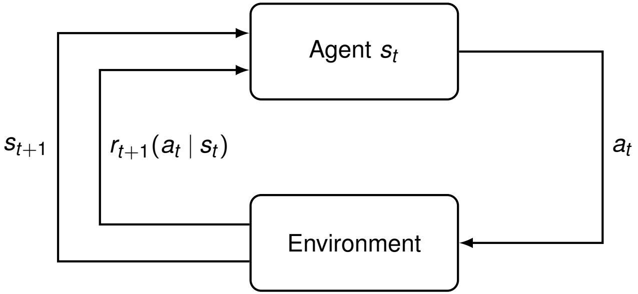

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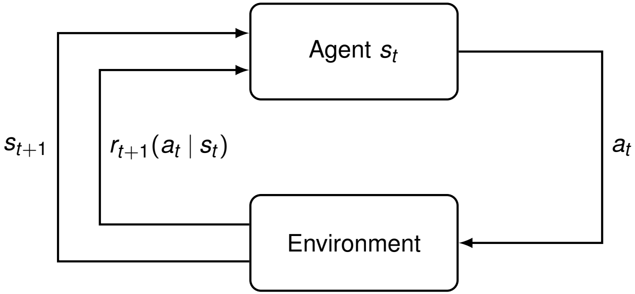

- maximizing the discounted cumulative reward

$$V(s_t) \equiv \sum_{k=0}^{\infty} \gamma^k \cdot r_{t+1+k}(a_{t+k} \mid s_{t+k}), \text{ where } 0 < \gamma < 1$$

Reinforcement Learning

Q-Learning [Watkins 1989]

- learns optimal action selection policy for any given Markov decision process

$$Q'(s,a) \quad = \quad (1-\alpha)\cdot Q(s,a)+\alpha\cdot\left(r(s,a)+\gamma\cdot V(s')\right) \text{ for a learning rate } \alpha\in[0,1]$$

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Reinforcement Learning

Maximize reward by learning importance sampling online

- radiance integral equation

$$L(x,\omega) = L_e(x,\omega) + \int_{\mathcal{S}_+^2(x)} f_s(\omega_i,x,\omega)\cos\theta_i \; L(h(x,\omega_i),-\omega_i) \; d\omega_i$$

NVIDIA.

Reinforcement Learning

Maximize reward by learning importance sampling online

- structural equivalence of integral equation and Q-learning

$$
\begin{aligned}
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- graphics example: learning the incident radiance

$$
Q'(x,\omega) = (1-\alpha)Q(x,\omega)+\alpha\left(L_e(y,-\omega)+\int_{\mathscr{S}_+^2(y)} f_s(\omega_i,y,-\omega)\cos\theta_i Q(y,\omega_i)d\omega_i\right)
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to be used as a policy for selecting an action ω in state x to reach the next state $y := h(x,\omega)$

- the learning rate α is the only parameter left

▶ Technical Note: Q-Learning

Reinforcement Learning

Online algorithm for guiding light transport paths

Function *pathTrace(camera, scene)*

```
throughput ← 1
ray ← setupPrimaryRay(camera)
for i ← 0 to ∞ do
    y, n ← intersect(scene, ray)
    if isEnvironment(y) then
        return throughput · getRadianceFromEnvironment(ray, y)
    else if isAreaLight(y)
        return throughput · getRadianceFromAreaLight(ray, y)
    ω, p_ω, f_s ← sampleBsdf(y, n)
    throughput ← throughput · f_s · cos(n, ω) / p_ω
    ray ← y, ω
```

Reinforcement Learning

Online algorithm for guiding light transport paths

Function *pathTrace(camera, scene)*

$throughput \leftarrow 1$

$ray \leftarrow \text{setupPrimaryRay}(camera)$

for $i \leftarrow 0$ **to** ∞ **do**

$y, n \leftarrow \text{intersect}(scene, ray)$

if $i > 0$ **then**

$$Q'(x,\omega) = (1-\alpha)Q(x,\omega) + \alpha \left(L_e(y,-\omega) + \int_{\mathscr{S}^2_+(y)} f_s(\omega_i, y, -\omega) \cos\theta_i \, Q(y,\omega_i) d\omega_i \right)$$

if *isEnvironment(y)* **then**

return $throughput \cdot \text{getRadianceFromEnvironment}(ray, y)$

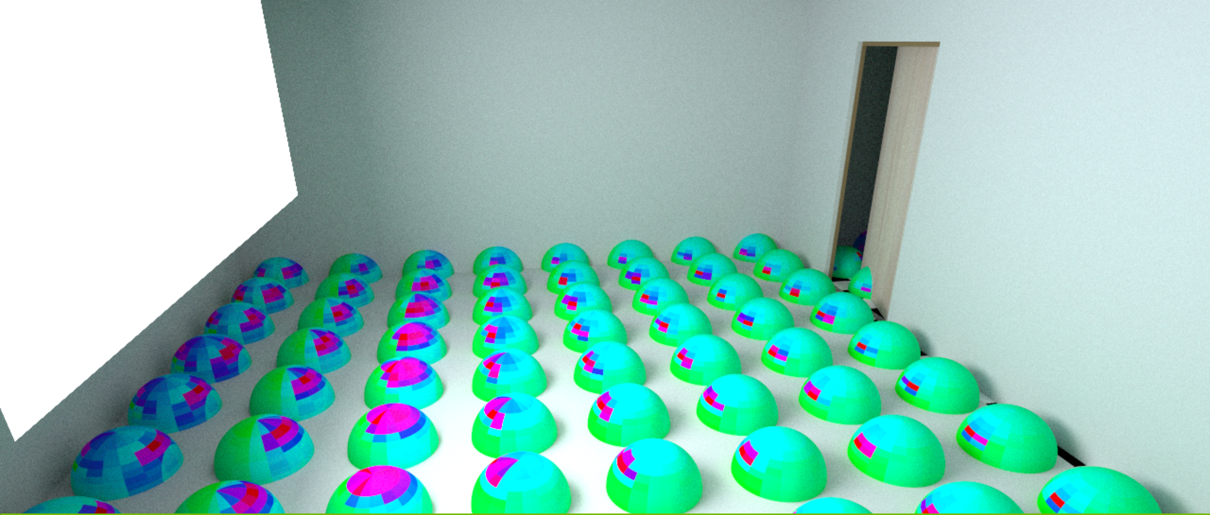
else if *isAreaLight(y)*

return $throughput \cdot \text{getRadianceFromAreaLight}(ray, y)$

$\omega, p_\omega, f_s \leftarrow$ **sampleScatteringDirectionProportionalToQ(y)**

$throughput \leftarrow throughput \cdot f_s \cdot \cos(n,\omega) \,/\, p_\omega$

$ray \leftarrow y,\ \omega$



approximate solution Q stored on discretized hemispheres across scene surface

2048 paths traced with BRDF importance sampling in a scene with challenging visibility

Path tracing with online reinforcement learning at the same number of paths

Metropolis light transport at the same number of paths

Reinforcement Learning

Guiding paths to where the value Q comes from

- shorter expected path length
- dramatically reduced number of paths with zero contribution
- very efficient online learning by learning Q from Q

Reinforcement Learning

Guiding paths to where the value Q comes from

- shorter expected path length
- dramatically reduced number of paths with zero contribution
- very efficient online learning by learning Q from Q
- directions for research
  - representation of value Q: data structures from games
  - importance sampling proportional to the integrand, i.e. the product of policy $\gamma \cdot \pi$ times value Q

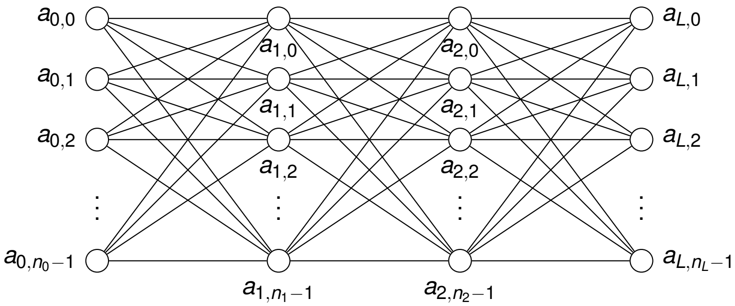
▶ On-line learning of parametric mixture models for light transport simulation
▶ Product importance sampling for light transport path guiding
▶ Fast product importance sampling of environment maps
▶ Learning light transport the reinforced way
▶ Practical path guiding for efficient light-transport simulation

From Graphics back to Machine Learning

Artificial Neural Networks in a Nutshell

Supervised learning of high dimensional function approximation

- input layer a_0, $L-1$ hidden layers, and output layer a_L



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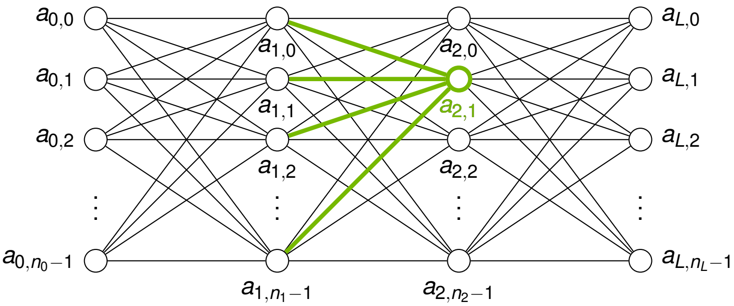

  - n_l rectified linear units (ReLU) $a_{l,i} = \max\{0, \sum w_{l,j,i} a_{l-1,j}\}$ in layer l

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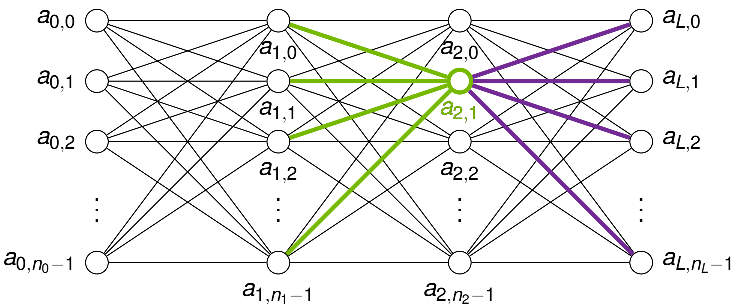

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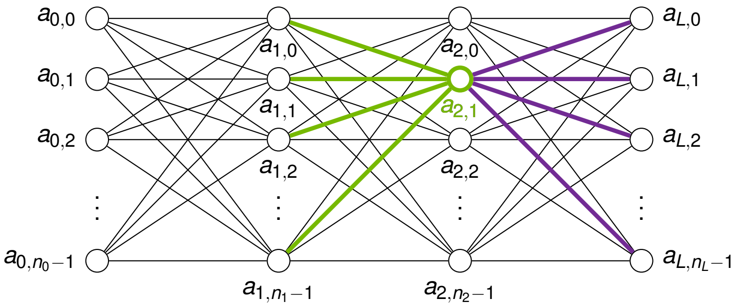

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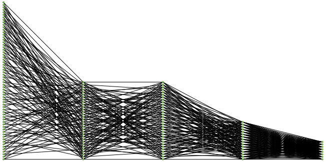
  - backpropagating the error $\delta_{l-1,i} = \sum_{a_{l,j}>0} \delta_{l,j} w_{l,j,i}$, update weights $w'_{l,j,i} = w_{l,j,i} - \lambda \delta_{l,j} a_{l-1,i}$ if $a_{l,j} > 0$

Artificial Neural Networks in a Nutshell

Supervised learning of high dimensional function approximation

- example architectures



classifier

▶ Multilayer feedforward networks are universal approximators

▶ Approximation capabilities of multilayer feedforward networks

▶ Universal approximation bounds for superpositions of a sigmoidal function

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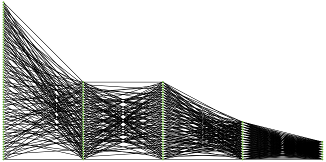

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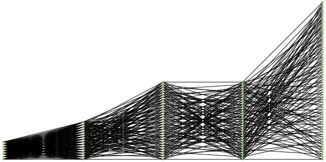

generator

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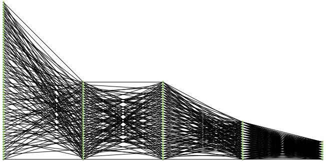

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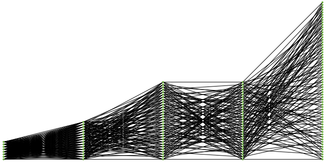

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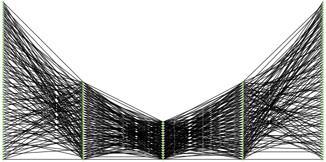

auto-encoder

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Efficient Training of Artificial Neural Networks

Using an integral equation for supervised learning

- Q-learning

$$Q'(x,\omega) = (1-\alpha)Q(x,\omega) + \alpha\left(L_e(y,-\omega) + \int_{\mathcal{S}_+^2(y)} f_s(\omega_i, y, -\omega)\cos\theta_i Q(y,\omega_i)d\omega_i\right)$$

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for $\alpha = 1$ yields the residual, i.e. loss

$$\Delta Q := Q(x,\omega) - \left(L_e(y,-\omega) + \int_{\mathcal{S}^2_+(y)} f_s(\omega_i, y, -\omega)\cos\theta_i Q(y,\omega_i)d\omega_i\right)$$

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  - light transport paths generated by a low discrepancy sequence for online training

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- supervised learning algorithm
  - light transport paths generated by a low discrepancy sequence for online training
  - learn weights of an artificial neural network for $Q(x,n)$ by back-propagating loss of each path

▶ A machine learning driven sky model

▶ Global illumination with radiance regression Functions

▶ Machine learning and integral equations

▶ Neural importance sampling

Efficient Training of Artificial Neural Networks

Learning from noisy/sampled labeled data

- find set of weights θ of an artificial neural network f to minimize summed loss L
  - using clean targets y_i and data $\hat{x}_i$ distributed according to $\hat{x} \sim p(\hat{x}|y_i)$

$$\text{argmin}_\theta \sum_i L(f_\theta(\hat{x}_i), y_i)$$

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$$\text{argmin}_\theta \sum_i L(f_\theta(\hat{x}_i), \hat{y}_i)$$

    - allows for much faster training of artificial neural networks used in simulations

- amounts to learning integration and integro-approximation

▶ Noise2Noise: Learning image restoration without clean data

Example Applications of Artificial Neural Networks in Rendering

Learning from noisy/sampled labeled data

- denoising quasi-Monte Carlo rendered images

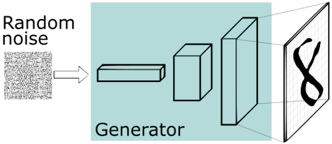
(a) Input (64 spp), 23.93 dB (b) Noisy targets, 32.42 dB (c) Clean targets, 32.95 dB (d) Reference (131k spp)

  - noisy targets computed 2000× faster than clean targets

Example Applications of Artificial Neural Networks in Rendering

Sampling according to a distribution given by observed data

- generative adversarial network (GAN)



▶ image source

▶ Tutorial on GANs

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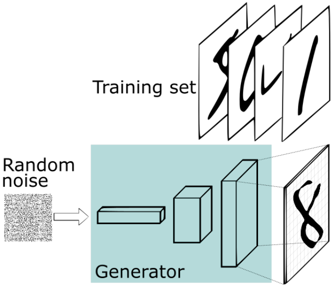

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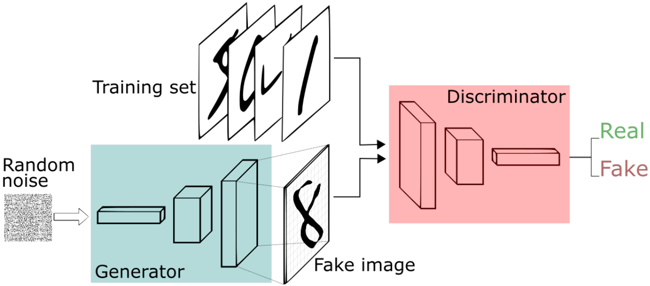

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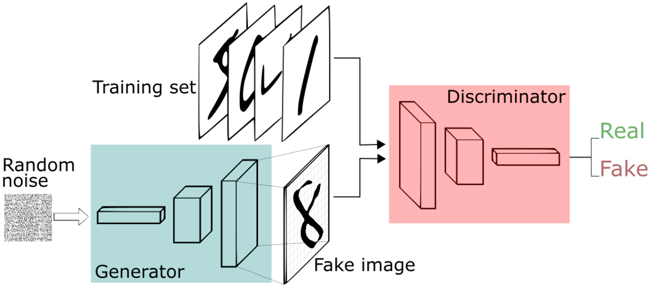
Example Applications of Artificial Neural Networks in Rendering

Sampling according to a distribution given by observed data

- generative adversarial network (GAN)

  - update generator G using $\nabla_{\theta_g} \sum_{i=1}^{m} \log(1 - D(G(\xi_i)))$



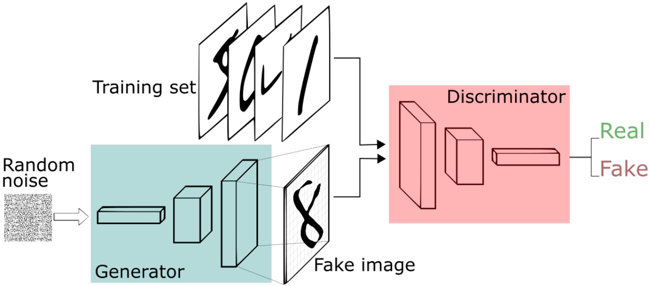
▶ image source

▶ Tutorial on GANs

Example Applications of Artificial Neural Networks in Rendering

Sampling according to a distribution given by observed data

- generative adversarial network (GAN)
  - update discriminator D (k times) using $\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} [\log D(x_i) + \log(1 - D(G(\xi_i)))]$
  - update generator G using $\nabla_{\theta_g} \sum_{i=1}^{m} \log(1 - D(G(\xi_i)))$



► image source
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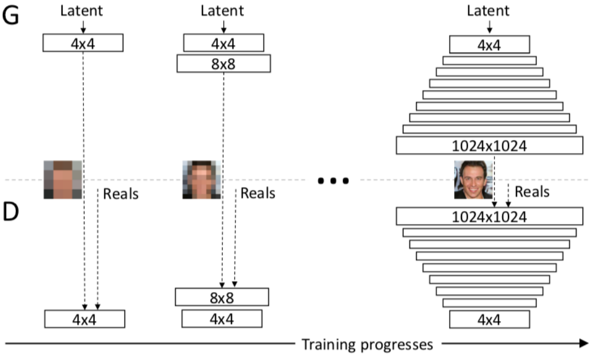
Example Applications of Artificial Neural Networks in Rendering

Sampling according to a distribution given by observed data

- Celebrity GAN

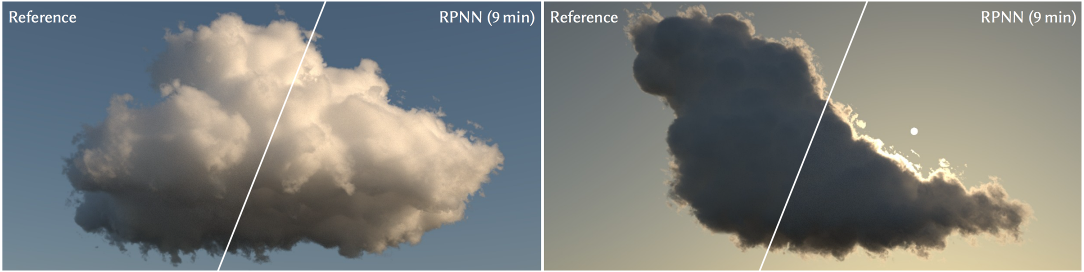


▶ Progressive growing of GANs for improved quality, stability, and variation

Example Applications of Artificial Neural Networks in Rendering

Replacing simulations by learned predictions for more efficiency

- much faster simulation of participating media
  - hierarchical stencil of volume densities as input to the neural network



► Deep scattering: Rendering atmospheric clouds with radiance-predicting neural networks

► Learning particle physics by example: Accelerating science with generative adversarial networks

Neural Networks linear in Time and Space

Neural Networks linear in Time and Space

Complexity

- the brain
  - about 10^{11} nerve cells with to up to 10^4 connections to others

Neural Networks linear in Time and Space

Complexity

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  - number of neural units

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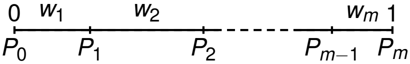
$$n_w = \sum_{l=1}^{L} c \cdot n_l = c \cdot n$$

  - constrain to constant number c of weights per neuron to reach complexity linear in n

Neural Networks linear in Time and Space

Sampling proportional to the weights of the trained neural units

- partition of unit interval by sums $P_k := \sum_{j=1}^{k} |w_j|$ of normalized absolute weights



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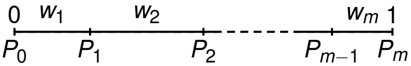

  - using a uniform random variable $\xi \in [0,1)$ to

    select input $i \Leftrightarrow P_{i-1} \leq \xi < P_i$ satisfying $\text{Prob}(\{P_{i-1} \leq \xi < P_i\}) = |w_i|$

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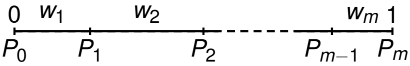

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- in fact derivation of quantization to ternary weights in $\{-1, 0, +1\}$
  - integer weights result from neurons referenced more than once
  - relation to drop connect and drop out

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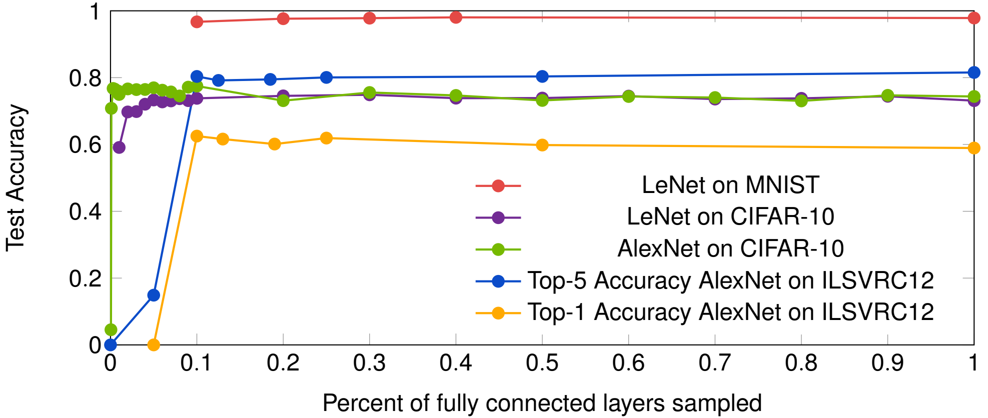
Sampling proportional to the weights of the trained neural units



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Sampling paths through networks

- complexity bounded by number of paths times depth L of network

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  - backwards random walks using sampling proportional to the weights of a neuron
  - compression and quantization by importance sampling

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Sampling paths through networks

- complexity bounded by number of paths times depth L of network

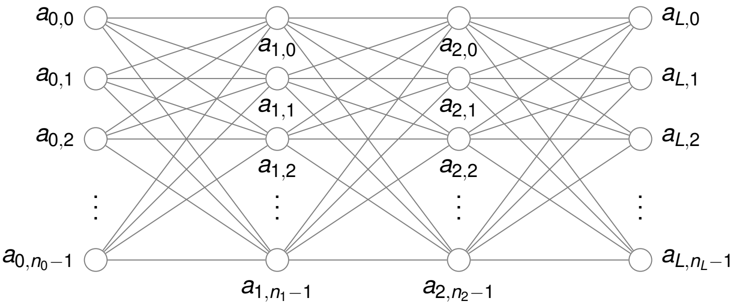
- application after training
  - backwards random walks using sampling proportional to the weights of a neuron
  - compression and quantization by importance sampling

- application before training
  - uniform (bidirectional) random walks to connect inputs and outputs
  - sparse from scratch

Neural Networks linear in Time and Space

Sampling paths through networks

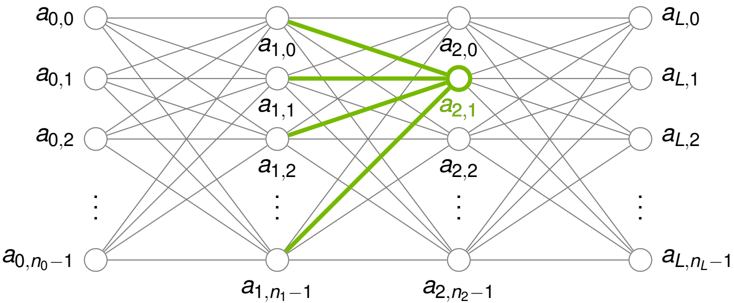
- sparse from scratch



Neural Networks linear in Time and Space

Sampling paths through networks

- sparse from scratch



Neural Networks linear in Time and Space

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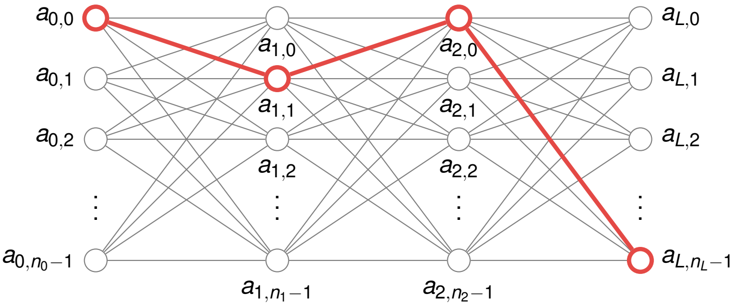


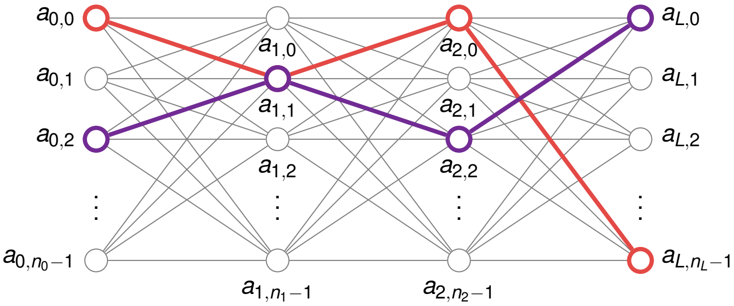
  - guaranteed connectivity

▶ Monte Carlo methods and neural networks

Neural Networks linear in Time and Space

Sampling paths through networks

- sparse from scratch



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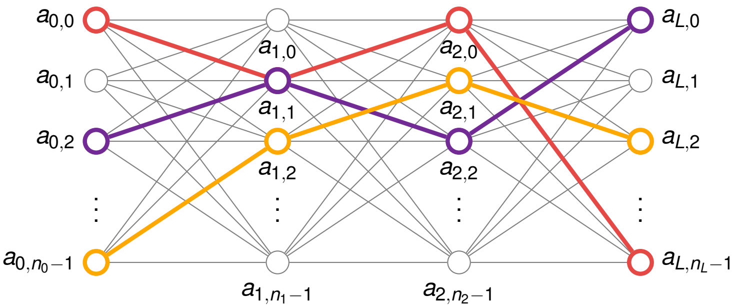

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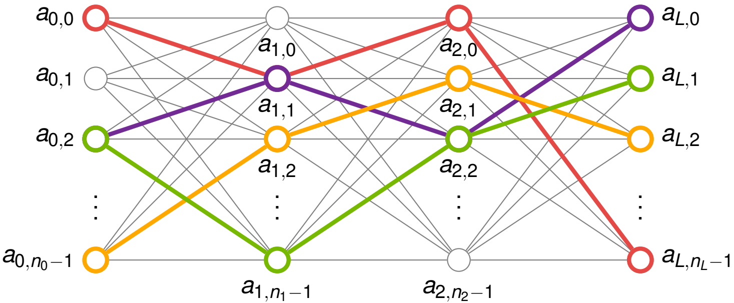

  – guaranteed connectivity

▸ Monte Carlo methods and neural networks

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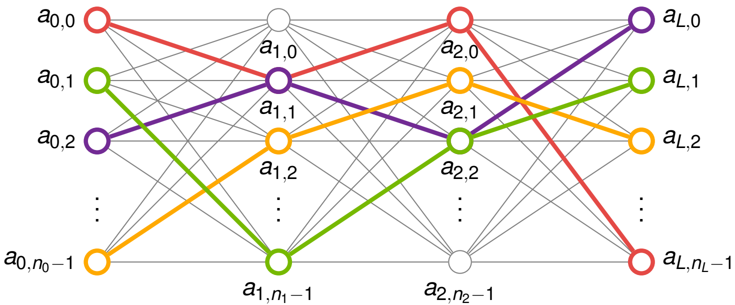
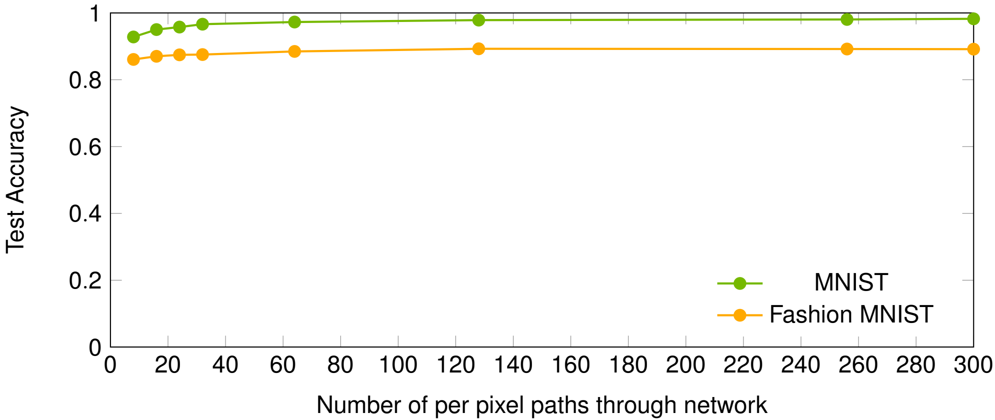


  - guaranteed connectivity and coverage

▶ Monte Carlo methods and neural networks

Neural Networks linear in Time and Space

Test accuracy for 4 layer feedforward network (784/300/300/10) trained sparse from scratch



From Machine Learning to Graphics and back

Summary

- light transport and reinforcement learning described by same integral equation
  - learn where radiance comes from
- neural networks results of linear complexity by path tracing
  - ternarization and quantization of trained artificial neural networks
  - sparse from scratch training